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Microwave applications of single negative materials

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Abstract

Metamaterials are reviewed first according to two different standpoints. One is rather old way of classification, which pays attention to the tensor property of the material index and magnetoelectric interaction. The other one has an eye to the sign of the material index, shifting its viewpoint rather to application. Double negative materials that have both negative permittivity and permeability have attracted microwave engineers' interest because of its novel characteristics. Though the present article mentions its application a little, the emphasis is on the behavior of the surface wave along the periphery of a single negative material. Potential application to a directional coupler and a resonator will be discussed in some detail.

Keywords: Composites; Dielectric properties; Electrical properties; Magnetic properties; Negative ε and μ

1. Introduction

Artificial materials (metamaterials) are considered to expand the horizon of microwave dielectric or magnetic material. In fact, they show extremely interesting properties, i.e. gigantic permittivity, big and arbitrarily controllable anisotropy, negative permittivity or permeability, coupling of electric and magnetic fields (magnetoelectric interaction) and so on. Those properties could be utilized for improving and developing microwave circuit and antenna components.

We will review the recent study of the metamaterials roughly, reorganizing the research results from two different standpoints. Then we will present our study on the single negative material. It has a negative value either for permittivity or permeability, which usually hampers the electromagnetic wave's propagation like the ionosphere. But careful examination of the boundary value problem reveals that the surface wave propagates along the boundary of the negative and positive index materials. This report, thus, summarizes its properties and addresses the possibility of new applications.

2. What is metamaterial?

The metamaterial is an artificial composite material fabricated mechanically, aligning small particles without chemical reactions. Since it has a long history of investigation, there are proposed quite many types of structure and ingredient. We are going to sort them in two different ways; first, in terms of the mutual relation of the electric and magnetic field in the constitutive equation; secondly, in terms of the sign of the permittivity and permeability.

2.1. Classification based on mutual relation of electric and magnetic field[1,2]

The constitutive relations of dielectric or magnetic materials are, in their simplest form, given by

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}, \quad \boldsymbol{B} = \mu \boldsymbol{H}. \tag{1}$$

But in the more complex materials, the relations include mixing terms, that is,

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} + \xi \boldsymbol{H}, \quad \boldsymbol{B} = \mu \boldsymbol{H} + \zeta \boldsymbol{E}. \tag{2}$$

These coefficients can be a tensor according to the anisotropy of the structure. Four cases are discriminated in Table 1, one of which is again divided into four cases as shown in Table 2. Since some of them are not realized in practice, but are virtual, fabrication principles are illustrated in Fig. 1 only for realistic examples.

The chiral media has a long history of study that originates from Pasteur in 1850s. Its main feature is the rotation of the polarization of EM waves and was first demonstrated in the microwave region by Lindman in 1914.[1] The application could

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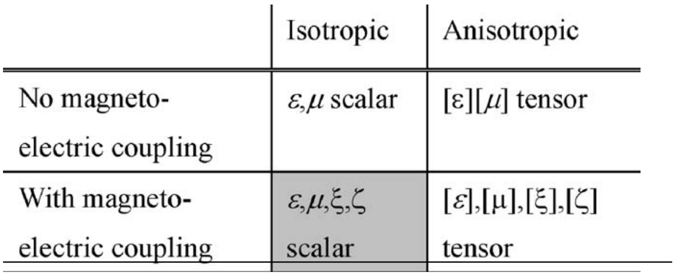
Table 1
Classification of bi-anisotropic material (hatched part is further classified in Table 2)



	Isotropic	Anisotropic
No magneto-electric coupling	ε,μ scalar	$[\varepsilon][\mu]$ tensor
With magneto-electric coupling	▨ $\varepsilon,\mu,\xi,\zeta$ scalar	$[\varepsilon],[\mu],[\xi],[\zeta]$ tensor

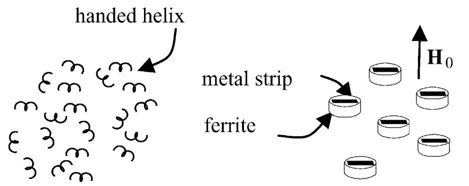
Table 2
Classification of bi-isotropic material (hatched part of Table 1 is classified)

	Non-chiral	Chiral
Reciprocal	Simple isotropic	Pasteur
Non-reciprocal	Tellegen	Bi-isotropic

be a polarizer, wave absorber radome and phase shifter.[1,3–5] But there have been no practical use so far. Other complex materials in Tables 1 and 2 have been studied mainly theoretically without much trial to apply in the real world.

2.2. Classification based on positive or negative material index

Combination of permittivity and permeability is illustrated in Fig. 2. Double positive material includes the conventional natural material as well as artificial dielectrics. Setting aside the natural materials, the latter has also been studied long. It was started 60 years ago aiming at lens antennas. Though it is not used in practice either, new application for microwave filters has been proposed by our group recently. But this topic will be omitted from the present paper.



(a) Pasteur material (b) Bianisotropic and nonreciprocal

Fig. 1. How to realize typical bi-isotropic and bi-anisotropic materials: (a) Pasteur material and (b) bi-anisotropic and non-reciprocal material.

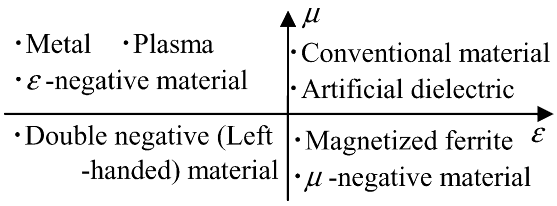


Fig. 2. Classification of materials in terms of sign of material index.

A single negative index material has a negative value either for permittivity or permeability. It shows the cut off characteristic for the plane wave because the wave number becomes pure imaginary. But if one has negative values for both permittivity and permeability, a plane wave propagates in the medium. It was analyzed by Veselago 40 years ago with little interest of the community.[6] He pointed out various novel properties of the wave propagation, which include backward wave propagation, negative refraction angle at the interface with double positive materials, focusing without diffraction, reverse Doppler and Cerenkov effect. Recent discovery of fabrication methods of the double negative material by Pendry revived the interest in the new metamaterials, pushing variety of proposal for the microwave applications.

3. How is negative index material realized?

3.1. Resonance of metal unit particles

Pendry's methods simply rely on the resonance of conducting metal wire.[7] Electrons in a thin conducting wire resonate with the parallel electric field and gives negative effective permittivity under the plasma frequency given by

$$\omega_\mathrm{p}^2 = \frac{n_\mathrm{eff} e^2}{\varepsilon_0 m_\mathrm{eff}}, \tag{3}$$

where

$$m_\mathrm{eff} = \frac{\mu_0 \pi r^2 e^2 n}{2\pi} \ln\frac{a}{r}\, n_\mathrm{eff} = n\frac{\pi r^2}{a^2}. \tag{4}$$

r is the radius of a wire, a the dimension of unit lattice, e the electron charge, and n the density of electrons in the conductor. A typical frequency characteristic is shown for the permittivity of a thin wire of square cross section in Fig. 3. The other implementations were also proposed using a loop wire[8] and capacitively loaded strip[9] which should be more realistic in fabrication, although under the same principle as Pendry's original proposal.

Pendry also proposed an effective method to realize a negative permeability utilizing the resonance of coupled open ring

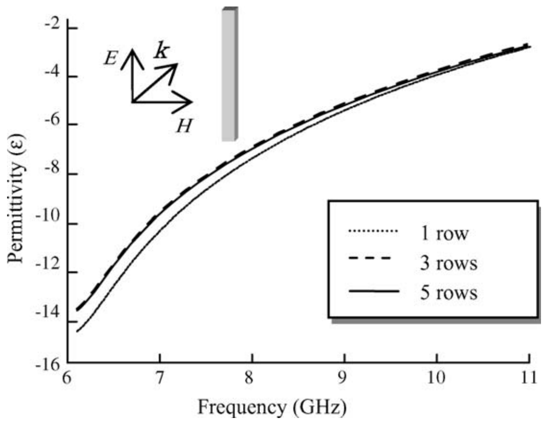


Fig. 3. Permittivity of metamaterial made of thin wires versus frequency. (Number of rows is counted to the longitudinal direction).

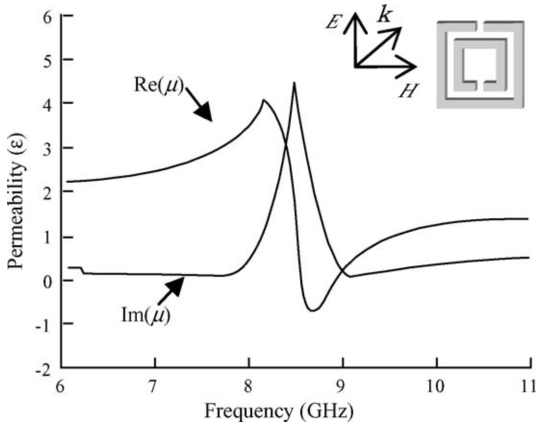


Fig. 4. Permeability of metamaterial made of coupled open ring resonators versus frequency.

resonator made of conducting metals.[10] A typical structure and its frequency characteristic is shown in Fig. 4. Because of the conductor loss at the resonance, the region of negative permittivity is quite narrow. When a double negative material is required, both structures mentioned above are combined with the operating frequency arranged to the same value.

3.2. Alignment of LC elements

Considering the operating frequency band is quite narrow due to the resonance behavior of μ-negative material, a new structure without resonance was investigated and proposed by three groups at the same time.[11–13] It is a combination of lumped-element inductors and capacitors, featuring a high-pass transmission line. In a conventional transmission line, a series inductance and a shunt capacitance is cascaded as shown in #1 of Fig. 5. Comparing the phase constant and the characteristic impedance with those of TEM mode, we find the correspondence

$$\mu = \frac{L}{a}, \quad \varepsilon = \frac{C}{a}. \tag{5}$$

Then the similar correspondence for the cascaded circuit with the capacitance and inductance interchanged holds as shown in #4 of Fig. 5.

$$\mu = -\frac{1}{\omega^2 Ca}, \quad \varepsilon = -\frac{1}{\omega^2 La}. \tag{6}$$

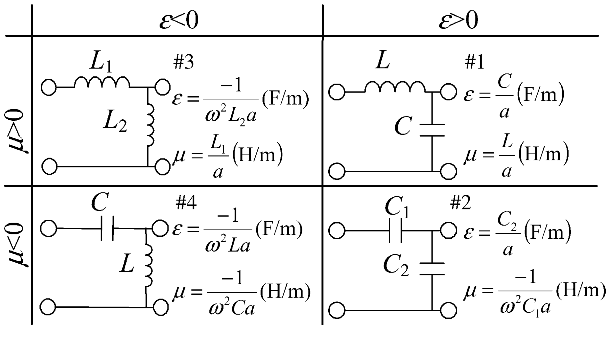


Fig. 5. Metamaterials based on lumped capacitors and inductors.

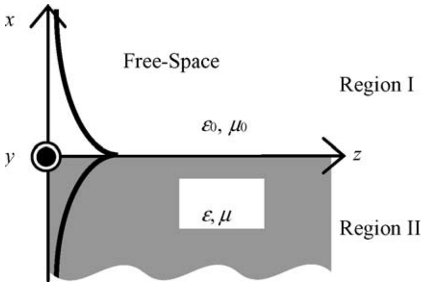


Fig. 6. Semi-infinite single negative material and surface wave along its boundary.

Table 3
Existence of surface wave for material parameter combinations

	$\varepsilon > 0$, $\mu < 0$	$\varepsilon > 0$, $\mu < 0$
TE wave ($E_z = 0$)	○	×
TM wave ($H_z = 0$)	×	○

Now, we find Eq. (6) exhibits a double negative material, showing that the circuit in #4 of Fig. 5 behaves like a TEM mode transmission line made of the medium with the material constant given by Eq. (6).

Fig. 5 gives the summary of the *LC* circuits that are equivalent to positive or negative index materials. Since they do not rely on the resonance of circuit elements, the operating frequency band is essentially wide, being different from the Pendry's structure. Thus, the double negative materials of these structures are finding many kinds of application for the microwave components, such as directional couplers, hybrids, resonators and antennas.

4. Wave propagation in single negative material

The propagation constant for a plane wave takes imaginary value in a single negative material, and hence the waves become evanescent. But the semi-infinite boundary with a double positive material maintains a surface wave, which is known as the surface plasmon for $\varepsilon < 0$ or the magnetostatic surface wave for $\mu < 0$. We reorganize those waves in the light of metamaterial application.

Consider the semi-infinite boundary of free space and a single negative material as shown in Fig. 6. Assuming the time and space dependence of the EM fields as $\exp\{j(\omega t - \beta z)\}$,we can solve the boundary value problem to obtain two different combinations for the surface wave propagation shown in Table 3.[14]

Since the surface waves above decay toward both perpendicular directions from the boundary, we can presume that a slab of finite thickness would sustain two surface waves along its both boundaries. It turned out true, analyzing the structure in the similar manner with the semi-infinite configuration.

If two waves propagate at the same time with the same propagation constant, they couple each other according to the overlap integral of both fields and exchange energy each other propagating along the z-axis. The rate of energy exchange is proportionate to the difference of β for both modes as is common in the coupled mode theory. An example of energy exchange between two

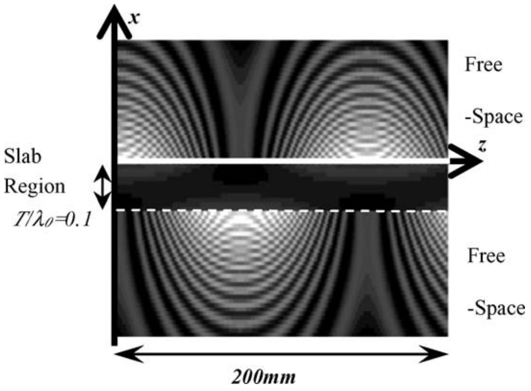


Fig. 7. Exchange of H_z field between two surfaces of slab ($T/\lambda_0 = 0.1$, $\varepsilon/\varepsilon_0 = 1.144$, $\mu/\mu_0 = -6.071$, $f = 13.6\,\mathrm{GHz}$).

surface modes is depicted in Fig. 7, suggesting an application to a compact microwave directional coupler.

5. Surface wave resonator

A ring surface wave resonator could be fabricated, if an island of single negative-index material is surrounded by a double positive material. In order to clarify the feasibility, we have studied the structure shown in Fig. 8 first, which is made of adjacent two-dimensional μ-negative material and a double positive material. The equivalent ε and μ are given by the value of lumped element per unit length as shown in Fig. 8(b) and (c). Applying the boundary condition for the tangential electric and magnetic field at the interface of both materials, we obtain the dispersion equation:

$$\beta = \frac{\omega}{a}\sqrt{\frac{L'_{\mathrm{pp}}(L'_{\mathrm{np}} - (1/\omega^2 C'_{\mathrm{np}}))\{C'_{\mathrm{ps}}(L'_{\mathrm{np}} - (1/\omega^2 C'_{\mathrm{np}})) - C'_{\mathrm{ns}}L'_{\mathrm{pp}}\}}{(L'_{\mathrm{np}} - (1/\omega^2 C'_{\mathrm{np}}))^2 - L'^2_{\mathrm{pp}}}} \tag{7}$$

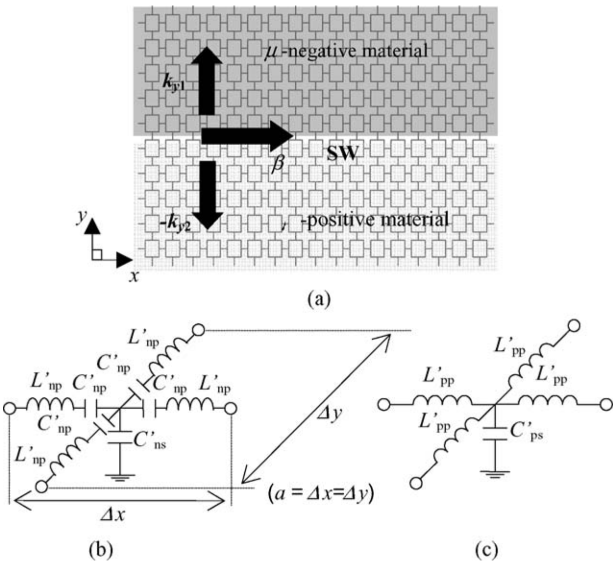


Fig. 8. Boundary of μ-negative and double positive materials for surface wave propagation. (a) Structure; (b) construction of μ-negative material by capacitors and inductors; and (c) construction of double positive material by capacitors and inductors.

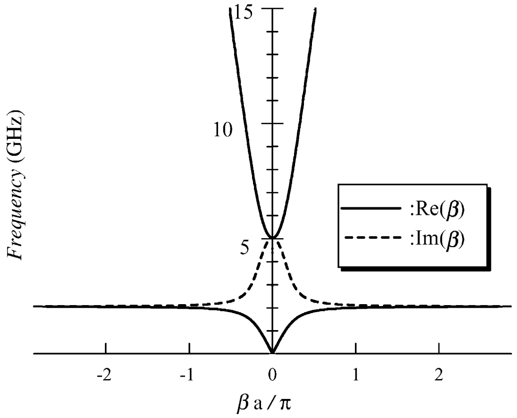


Fig. 9. Propagation constant vs. frequency for surface wave in structure of Fig. 8.

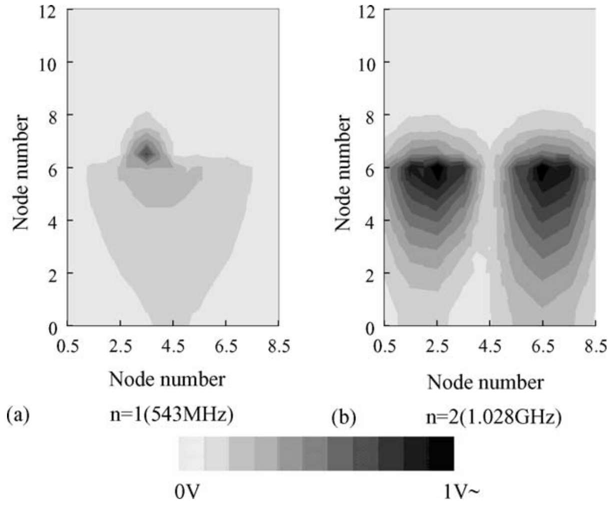


Fig. 10. Resonance of surface wave in terminated structure of Fig. 8.

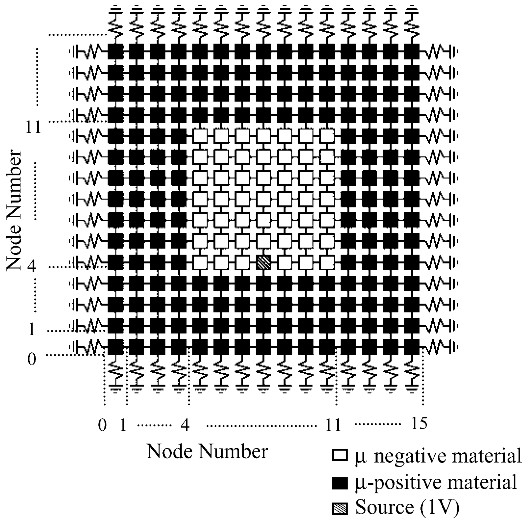
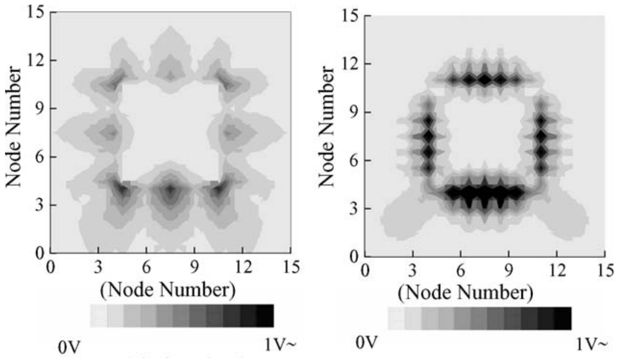


Fig. 11. Square patch resonator of μ-negative material embedded in double positive surrounding.



1.35GHz (n=4)

1.9GHz (n=10)

Fig. 12. Surface wave resonance of square patch resonator in Fig. 11.

which is described in Fig. 9 for the parameters given in the caption. If it is terminated by 50 Ω resistance at the upper and lower ends, while it is short-circuited at the right and left ends, it will make a one-dimensional surface wave resonator. The resonances occur showing integer number of voltage maximums in Fig. 10.

In Fig. 11, the square region from the node number 4 to 11 for both coordinates constitute a μ-negative area, while other region is occupied by a double positive material. We simulated the structure shown in Fig. 12 and had an evidence of square patch resonator operation. In fact, Fig. 12 shows 8 and 20 V maximums, respectively, for each excitation frequency along the periphery of the square μ-negative area.

6. Conclusion

This article has tried to introduce a seemingly tricky method to create a dielectric or magnetic material. Instead of molecules, it basically relies on metallic particles, which are aligned to make an assembled structure. Electromagnetic property of the structure is macroscopically defined, making it possible to extract the effective μ and ε.

More tricky measures are to let a cascaded *LC* circuit behave like a transmission line with negative material indices. This “material” looks more suspicious than the former. But the important thing for microwave engineers is a final circuit that behaves as if it were made of such and such material. As long as the characteristic of a circuit is the same as the designed value, they do not care the procedure.

These artificial materials are recently called “metamaterial”, using a word sounding exotic. It may be an expression of the strong hope for microwave engineers to develop new epoch-making circuits. Since its study has just started, in terms of application especially, they will try hard to open up the new vistas of microwave materials.

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